A Correlation Based Method for Measuring and Monitoring the Impulse Response of Analog LTI Systems with low Realization Cost

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This article deals with the problem of measuring the impulse response of a linear time invariant (LTI) analog network. At the beginning three known measurement principles are discussed and compared with respect to their advantages and disadvantages. It is shown that good results can only be attained at the cost of measurement expense. Subsequently a new correlation based method is presented where low complexity and high quality results have not to be a contradiction. The measurement circuit basically consists of two m-sequence generators and an analog low pass filter. The characteristics of the proposed measurement circuit are examined and explained by two examples.

Categories and Subject Descriptors: System characteristics, digital measurement

Additional Key Words and Phrases: Impulse response, measurement, correlation, shift register, m-sequences

1 PRELIMINARY REMARK

The response $y(t)_t$ of a linear time invariant system when an input signal $x(t)$ is applied is given by Duhamel's well known convolution integral [1]

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h_0(t-\tau) d\tau.$$

h_0(t) is the impulse response, representing the system characteristics in time domain.

It may be noted here that the impulse response is of great importance for both solving theoretical as well as practical problems.

In the field of engineering $h_0(t)$ must often be determined by measurement. Which method to take? Generally speaking we can choose a time domain or a frequency domain based approach. We will discuss this point later on. The focus of this article will be on the presentation of a new low complexity time domain and correlation based measurement method.

Before proceeding it is to be expressed that the new method is suitable for linear systems with time invariance characteristic (LTI).

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2 WAYS TO MEASURE THE IMPULSE RESPONSE OF AN LTI SYSTEM

2.1 Impulse excitation method

When an LTI-system is excited by a small rectangular impulse of duration T_0 and amplitude A, the reaction $\tilde{h}_0(t)$ approximates (except of a constant factor c) the impulse response, i. e.

$$\tilde{h}_0(t) \approx c \cdot h_0(t) \tag{2}$$

where

$$c = AT_0. \tag{3}$$

When T_0 decreases the approximation quality increases. This is true because the rectangular impulse gets more similar to the Dirac impulse (and vice versa). As a consequence the amplitude decreases as well. This can be compensated by increasing A, however the increase must be stopped before internal system overflow or overload occurs, otherwise non-linearities would be introduced. The maximum allowed width of the input rectangular pulse depends on how fast the impulse response changes. Generally speaking in most cases $\tilde{h}_0(t)$ is more or less buried in the system noise.

As a consequence the impulse method is suitable only in some special cases (e. g. in low noise systems) because of its low noise immunity.

2.2 The spectral method

The convolution in time domain corresponds to a multiplication in frequency domain, i. e. the system characteristic can be expressed in terms of the quotient of the complex spectrum of output signal and input signal. There are some limitations when choosing the excitation signal. This detail is not be discussed here.

$$h_0(t) = \mathrm{IFT}\left\{\frac{\mathrm{FT}\{y(t)\}}{\mathrm{FT}\{x(t)\}}\right\} \tag{4}$$

(FT means Fourier-transformation and IFT the inverse Fourier-transformation).

The noise like phenomena are limited in this case.

The disadvantage of this method is the relatively large amount of technical build up needed to realize equ. (4): An AD-converter, a clock oscillator and a signal processor or additional hardware for realizing the Fourier transform.

2.3 The correlation method

The measurement set up is depicted in fig. 1. It consists of a signal source with excitation signal $x(t)$, a delay line (delay τ), the DUT (Device Under Test) with impulse response h_0(t), a multiplier and an averaging filter [3].

Presuming the DUT to be causal we have (the symbol $*$ designates convolution):

$$y(t) = x(t) * h_0(t) \tag{5}$$

$$= \int_0^\infty h_0(\sigma)x(t-\sigma)d\sigma \tag{6}$$

and

$$z(t) = \int_0^\infty h_0(\sigma)x(t-\sigma)x(t-\tau)d\sigma. \tag{7}$$

Averaging $z(t)$ results in

$$\bar{z}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T \int_0^\infty h_0(\sigma)x(t-\sigma)x(t-\tau)d\sigma\, dt \tag{8}$$

$$= \int_0^\infty \lim_{T\to\infty} \frac{1}{T} \int_0^T h_0(\sigma)x(t-\sigma)x(t-\tau)dt\, d\sigma \tag{9}$$

$$= \int_0^\infty h_0(\sigma) \underbrace{\lim_{T\to\infty} \frac{1}{T} \int_0^T x(t-\sigma)x(t-\tau)dt\, d\sigma}_{\varphi_{xx}(\tau-\sigma)} \tag{10}$$

$$= \int_0^\infty h_0(\sigma)\varphi_{xx}(\tau-\sigma)d\sigma. \tag{11}$$

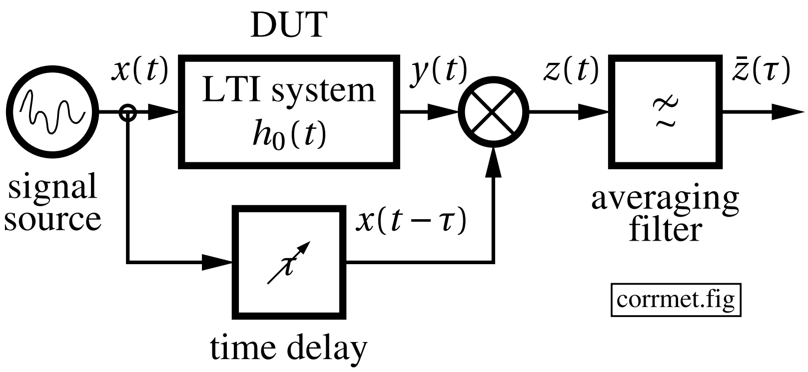

Figure 1: Setup for the determination of the impulse response based on the correlation method.

If the autocorrelation function $\varphi xx\ (\tau)$ is a small impulse with area F_φ centered around $\tau = 0$ and if $\varphi_{xx}\ (\tau) \equiv 0$ for $|\tau| \geq T_\varphi$[5] (the impulse is small, if $h_0(t) \approx h_0(t - T_\varphi)\ \forall\ t > T_\varphi$)), we get

$$\bar{z}(\tau) \approx h_0(\tau) \int_{-\infty}^{\tau} \varphi_{xx}(\sigma)d\sigma \tag{12}$$

$$\approx h_0(\tau)F_\varphi. \tag{13}$$

We conclude that the average value $\bar{z}(\tau)$ equals (apart from the constant F_φ) the impulse response.

By choosing an appropriate integration time for the averaging filter the noise contained in the output $\bar{z}(\tau)$ can be kept arbitrarily low. Of course a large integration time results in a long measurement cycle.

As can be seen using correlation for impulse response measurement we need

- an analog multiplier,
- a time delaying network with an *adjustable* delay time τ, i. e. impulse response $\delta_0(t-\tau)$.

In practice the implementation of both systems can be done only at a great expense. E. g. the time delaying system can be realized with good quality only based on digital components: An analog to digital converter, an addressable memory and a digital to analog converter.
This makes clear that the correlation based method as well as the spectral method both suffer from a relatively high realization complexity.

3 A MODIFIED CORRELATION BASED MEASUREMENT METHOD

Correlation based measurements are attractive because of their excellent noise immunity. To overcome the relatively high complexity of this method in the original form shown in fig. 1 some modifications are now proposed to reduce the hardware effort. We start with a modification of fig. 1, leading to the circuit depicted in fig. 2.

3.1 Substitution of the multiplier

The measurement set up in fig. 2 differs from that shown in fig. 1 by *two* sources with different signals $x_1(t)$ and $x_2(t)$, while the former one has only one signal. We now have

$$\bar{z}(\tau) = \int_0^\infty h_0(\sigma)\varphi_{x_2x_1}(\tau-\sigma)d\sigma \quad (14)$$

where $\varphi_{x_2x_1}(\tau)$ denotes the cross correlation function of $x_1(t)$ and $x_2(t)$ respectively. When choosing $x_2(t)$ as a *binary* function with values $x_2 \in \{a_2, b_2\}$, the multiplier degenerates into a switch and two coefficient elements. Fig. 3 shows the circuit. It is easy and simple to realize.

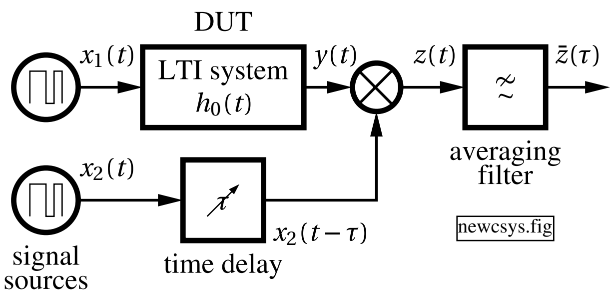

Figure 2: Measurement of the impulse response using the modified correlation method.

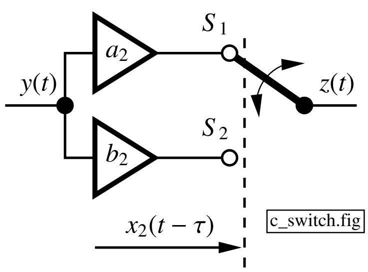

Figure 3: Substitution of the multiplier. The switch position is S_1 for $x_2(t-\tau) = a_2$ or S_2 if $x_2(t-\tau) = b_2$.

There are two cases which are of special interest from a practical point of view:

1. $a_2 = 1$ and $b_2 = 0$.
   The multiplier is substituted by an on-off-switch (SPST[1]). No need for coefficient elements here.
2. $a_2 = 1$ and $b_2 = -1$.
   In this case a switch is used in addition to an amplifier with amplification $V = -1$.

Both cases are easy to put into practice, an important point for a realization with a low expenditure.

3.2 Considering the choice of $x_1(t)$ and $x_2(t)$

Regarding the excitation signals $x_1(t)$ and $x_2(t)$ from fig. 2 we have the following requirements:

1. As already mentioned $x_2(t)$ should be a binary function to simplify the realization of the multiplier.
2. To achieve $\bar{z}(\tau)$approximates $h_0(\tau)$ sufficiently well, the cross correlation function $\varphi_{x_1x_2}(\tau)$ must resemble the Dirac impulse $\delta_0(\tau)$ as close as possible.
3. It would be attractive if the impulse response could be measured continuously and periodically (i. e. not only once). In this case slow trends caused by ageing of the system elements or by temperature influence would be easy to recognize. To realize such a periodization of the impulse response $\varphi_{x_1x_2}(\tau)$ must be periodic as well.

A suitable choice is to select $x_1(t)$ and $x_2(t)$ as m-sequences. A m-sequence can be realized very easily by using a shift register with an appropriate signal feedback[2] [4]. See fig. 4 for details.

The values 0/1 of the sequence $x(t)$ are mapped to a_1/b_1 to generate $x_1(t)$ resp. a_2/b_2 to generate $x_2(t)$ i. e. for $\nu \in \{1, 2\}$:

$$x_\nu(t) = (b_\nu - a_\nu)x(t) + a_\nu. \tag{15}$$

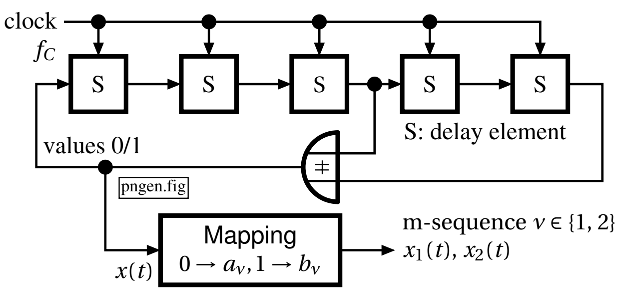

Figure 4: Example for a m-sequence generator. N = 5 delay elements S are used. The EX-OR gate serves as a modulo-2 adder.

[1] Single pole single throw.
[2] Certain taps of the shift register chain are fed to a modulo-2 adder whose output is connected with the shift register's input.

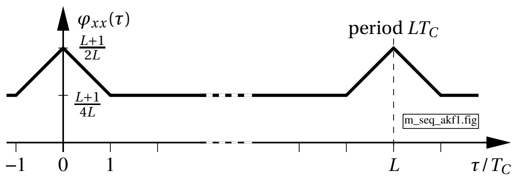

Figure 5: Autocorr. func. $\varphi_{xx}(\tau)$ of the m-sequence x(t).

For the following considerations the characteristics of the m-sequence are of interest:

1. $x(t), x_1(t)$ and $x_2(t)$ are periodic sequences with period $L = T_C$, where $L = 2^N$ and $T_C = 1/f_C$.
2. $x(t)$ has the autocorr. funct. depicted in fig. 5.
3. For the cross correlation $\varphi_{xx}(\tau)$ we get

$$\begin{aligned}\varphi_{x_2 x_1}(\tau) = (b_2 - a_2)(b_1 - a_1)\varphi_{xx}(\tau) + \\ + \overline{x}(\tau) a_1 (b_2 - a_2) + \\ + \overline{x}(\tau) a_2 (b_1 - a_1) + \\ + a_1 a_2.\end{aligned} \tag{16}$$

   The basic shape of the function $\varphi x_1 x_2(\tau)$ is sketched in fig. 6.
4. The functions can be represented by a Fourier series ($\nu \in \{1,2\}$):

$$x_\nu(t) = \sum_{\mu=-\infty}^{\infty} c_\mu^{(\nu)} e^{j2\pi\mu\frac{f_C}{L}t} \tag{17}$$

   where

$$|c_\mu^{(\nu)}| = \begin{cases} \frac{1}{2}\alpha_\nu + \frac{1}{2L}\beta_\nu & \text{for} \quad \mu = 0 \\ \frac{\sqrt{L+1}}{2L}\beta_\nu \left| \frac{\sin \pi \frac{\mu}{L}}{\pi \frac{\mu}{L}} \right| & \text{for} \quad \mu \neq 0 \end{cases} \tag{18}$$

   with substitution $\alpha_\nu = b_\nu + a_\nu$, $\beta_\nu = b_\nu - a_\nu$. It is $c_{(-\mu)}^{(\nu)} = c_\mu^{(\nu)*}$ because the sequences are real valued. In table 1 the Fourier coefficients for both special cases are listed.

The desired impulse like behaviour of the cross correlation function results if for given values a_2 and b_2 the values a_1 and b_1 are selected as follows:

$$b_1 = -a_1 \left(1 - \frac{4a_2}{(L+1)(a_2 + b_2)}\right). \tag{19}$$

Equation (19) results from the fact that the "offset" in fig. 6 is set to zero. For $a_2 = -1$ and $b_2 = 1$ the values $a_1 = 0$ and b_1 arbitrary, for $a_2 = 0$ and $b_2 = 1$ results $b_1 = -a_1$. In fig. 7 an example is given.

3.3 Elimination of the time delaying element

Up to now it is an open question how to realize the time delaying element in fig. 2. Below it is shown that this element is not required for the measurement of the impulse response. It can in fact be omitted using a little "trick".
If the two m-sequence generators are to be operated at slightly different clock frequencies the two sequences will – figuratively speaking –"slide" past each other. We achieve the same

effect as if the delay would increase (or decrease) permanently. Fig. 8 shows the realizing circuitry.

Before proceeding we must point to a general change concerning our measurement principle when using the new "sliding"method to substitute the time delay.

In the section 2.3 a fixed delay τ is adjusted, then the measurement process takes place and then τ was changed for the next measurement. Hence the measurement is done on a "step by step" base. τ remains constant for a certain time step. In contrast to that the measurement process using the "sliding" sequences is based on a continuously changed delay. As a consequence we do not get $\bar{z}(\tau)$ at the low pass output exactly. We get $w(t)$ instead, i. e. a signal differing more or less from the expected impulse response $h_0(t)$.

3.4 The output signal $w(t)$

The m-sequence time functions $x_1(t)$ and $x_2(t)$ are periodic with periods $T_1 = L/f_C$ and $T_2 = L/((1-\delta)\, f_C)$. For the Fourier series of $x_1(t)$ and $x_2(t)$ we get using $f_{P1} = 1/T_1 = f_C/L$ *and* $f_{P2} = 1/T_2 = (1-\delta)\, f_C/L$

$$x_1(t) = \sum_{\mu=-\infty}^{\infty} c_\mu^{(1)} e^{j2\pi\mu f_{P1} t} \tag{24}$$

$$x_2(t) = \sum_{\mu=-\infty}^{\infty} c_\mu^{(2)} e^{j2\pi\mu f_{P2} t}. \tag{25}$$

Assuming steady state at the output of the LTI-system excited by $x_1(t)$ the following time function $y(t)$ results:

$$y(t) = \sum_{\mu=-\infty}^{\infty} H(j2\pi\mu f_{P1}) c_\mu^{(1)} e^{j2\pi\mu f_{P1} t}. \tag{26}$$

We now form

$$z(t) = y(t) \cdot x_2(t) \tag{27}$$

$$= \left(\sum_{\mu=-\infty}^{\infty} H(j2\pi\mu f_{P1}) c_\mu^{(1)} e^{j2\pi\mu f_{P1} t} \right) \cdot \left(\sum_{\nu=-\infty}^{\infty} c_\nu^{(2)} e^{j2\pi\nu f_{P2} t} \right). \tag{28}$$

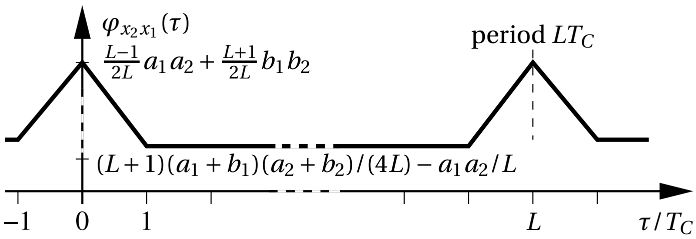

Figure 6: Cross correlation function $\varphi_{x_2 x_1}(\tau)$.

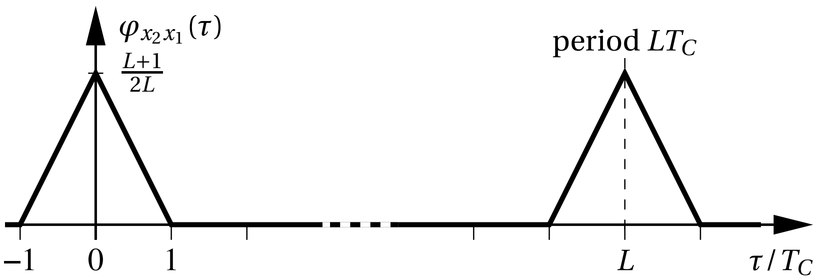

Figure 7: Cross correlation function $\varphi_{x_2 x_1}(\tau)$ for the special case $a_2 = 0$, $b_2 = 1$, $a_1 = -1$ and $b_1 = 1$.

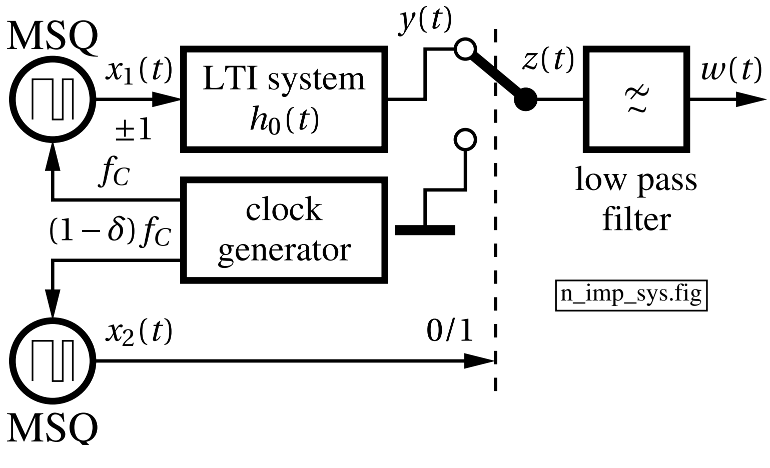

Figure 8: The proposed method for measuring the impulse response with $a_1 = -1$, $b_1 = 1$, $a_2 = 0$ and $b_2 = 1$. Used are two m-sequence generators MSQ (sequence length L) with different clocking, a clock oscillator, a SPST switch and a low pass filter.

Case 1: $a_1 = -1, b_1 = 1, a_2 = 0, b_2 = 1$
$$\lvert c_\mu^{(1)} \rvert = \begin{cases} \frac{1}{L} & \text{for} \quad \mu = 0 \\ \frac{\sqrt{L+1}}{L} \left\lvert \frac{\sin \pi \frac{\mu}{L}}{\pi \frac{\mu}{L}} \right\rvert & \text{for} \quad \mu \neq 0 \end{cases} \qquad (20)$$ and $$\lvert c_\mu^{(2)} \rvert = \begin{cases} \frac{1}{2}(1+\frac{1}{L}) & \text{for} \quad \mu = 0 \\ \frac{1}{2} \frac{\sqrt{L+1}}{L} \left\lvert \frac{\sin \pi \frac{\mu}{L}}{\pi \frac{\mu}{L}} \right\rvert & \text{for} \quad \mu \neq 0. \end{cases} \qquad (21)$$

Case 2: $a_1 = 0, b_1 = 1, a_2 = -1, b_2 = 1$
$$\lvert c_\mu^{(1)} \rvert = \begin{cases} \frac{1}{2}(1+\frac{1}{L}) & \text{for} \quad \mu = 0 \\ \frac{1}{2} \frac{\sqrt{L+1}}{L} \left\lvert \frac{\sin \pi \frac{\mu}{L}}{\pi \frac{\mu}{L}} \right\rvert & \text{for} \quad \mu \neq 0. \end{cases} \qquad (22)$$ and $$\lvert c_\mu^{(2)} \rvert = \begin{cases} \frac{1}{L} & \text{for} \quad \mu = 0 \\ \frac{\sqrt{L+1}}{L} \left\lvert \frac{\sin \pi \frac{\mu}{L}}{\pi \frac{\mu}{L}} \right\rvert & \text{for} \quad \mu \neq 0 \end{cases} \qquad (23)$$

Table 1: Fourier coefficients of the functions $x_1(t)$ and $x_2(t)$ for the special cases under consideration.

By rearranging the summation terms follows

$$z(t) = \sum_{\mu=-\infty}^{\infty} H(j2\pi\mu f_{P1}) c_\mu^{(1)} \cdot \\ \cdot \sum_{\nu=-\infty}^{\infty} c_\nu^{(2)} e^{j2\pi(\mu+\nu) f_{P1} t} e^{-j2\pi\delta\nu f_{P1} t}. \tag{29}$$

Because we expect low pass behaviour of the DUT, using $f_H = \lambda f_{P1} = \lambda f_C / L,\ \lambda \in \mathbb{N}$

$$z(t) = \sum_{\mu=-\lambda}^{\lambda} H(j2\pi\mu f_{P1}) c_\mu^{(1)} \cdot \\ \cdot \sum_{\nu=-\infty}^{\infty} c_\nu^{(2)} e^{j2\pi(\mu+\nu) f_{P1} t} e^{-j2\pi\nu\delta f_{P1} t}. \tag{30}$$

If the low pass filter used for averaging has a cut-off frequency f_g with

$$\lambda\delta f_{P1} \le f_g \le (1-\lambda\delta) f_{P1}, \tag{31}$$

then only such summation terms of eqn. (30) appear at the output, for which $\mu + \nu = 0$ holds. From this the time signal $w(t)$ results:

$$w(t) = \sum_{\mu=-\lambda}^{\lambda} c_{\mu}^{(1)} c_{-\mu}^{(2)} H(j2\pi\mu f_{P1}) e^{j2\pi\mu\delta f_{P1} t}. \tag{32}$$

If we put eqn. (20) and eqn. (21) resp. Glg. (22) and (23) into eqn. (32) it follows

$$w(t) = \frac{L+1}{2L^2} \sum_{\mu=-\lambda}^{\lambda} \left(\frac{\sin \pi \frac{\mu}{L}}{\pi \frac{\mu}{L}} \right)^2 \cdot \tag{33}$$

$$\cdot H(j2\pi\mu f_{P1}) e^{j2\pi\mu\delta f_{P1} t}. \tag{34}$$

When the period length L of the shift register sequence was chosen to hold

$$\left(\frac{\sin \pi \frac{\lambda}{L}}{\pi \frac{\lambda}{L}} \right)^2 \approx 1 \tag{35}$$

the calculation leads to

$$w(t) = \frac{L+1}{2L^2} \sum_{\mu=-\lambda}^{\lambda} H(j2\pi\mu f_{P1}) e^{j2\pi\mu\delta f_{P1} t}. \tag{36}$$

For eqn. (36) we can write

$$\begin{aligned} w(t) &= \frac{L+1}{2L^2} \sum_{\mu=-\infty}^{\infty} \left(\int_{-\infty}^{\infty} h_0(\tau) e^{-j2\pi\mu f_{P1}\tau} d\tau \right) \cdot \\ &\quad \cdot e^{j2\pi\mu\delta f_{P1} t} \\ &= \frac{L+1}{2L^2} \int_{-\infty}^{\infty} h_0(\tau) \sum_{\mu=-\infty}^{\infty} e^{-j2\pi\mu f_{P1}(\tau-\delta t)} d\tau \\ &= \frac{L+1}{2L^2 f_{P1}} \int_{-\infty}^{\infty} h_0(\tau) \sum_{\mu=-\infty}^{\infty} \delta_0(\tau - \delta t - \frac{\mu}{f_{P1}}) d\tau \\ &= \frac{L+1}{2L^2 f_{P1}} \sum_{\mu=-\infty}^{\infty} h_0(\delta t - \frac{\mu}{f_{P1}}) \\ &= \frac{L+1}{2L f_C} \sum_{\mu=-\infty}^{\infty} h_0(\delta t - \frac{\mu L}{f_C}). \end{aligned} \tag{37}$$

Summarizing result: At the low pass output the periodized, stretched and scaled (by an amplitude factor) impulse response of the DUT appears. The amplitude factor has the value $(L + 1)/(2Lf_C)$, the stretching factor is $\Gamma = 1/\delta$ and the period time is $\Gamma L/f_C = \Gamma L T_C$.

3.5 Dimensioning specification

Prior to dimensioning the following variables must be known:

- the cut off frequency f_H of the device under test,
- the length (duration) T_h of the impulse response,
- the desired spreading factor Γ.

The determination is as follows:

1. First of all it must be fixed, "how well" eqn. (35) is to be satisfied. For this the quotient $l_0 = L/\lambda$, must be chosen suitable.
2. With a known value for l_0 and known cut off frequency f_H of the DUT we get f_C from
   $f_C = l_0 f_H$ *(38)*

3. To avoid a temporal overlap of the impulse response in equ. (37),

$$\frac{L}{f_C} \geq T_h \tag{39}$$

must be satisfied. Because for the period length L there is the condition $L = 2^N - 1$, $N \in N$, results

$$N = \lceil \mathrm{ld}(f_C T_h + 1) \rceil \tag{40}$$

($\lceil x \rceil$: Largest integer not exceeding x).

4. The sequence length L now results from

$$L = 2^N - 1. \tag{41}$$

5. Now λ can be fixed:

$$\lambda = \left\lfloor \frac{L}{l_0} \right\rfloor \tag{42}$$

($\lceil x \rceil$: Smallest integer not falling below x).

6. Finally we need the cut off frequency f_g of the averaging low pass. It can be calculated as

$$\frac{\lambda}{\Gamma L} f_C \leq f_g \leq \left(\frac{1}{L} - \frac{\lambda}{\Gamma L} \right) f_C. \tag{43}$$

4 EXAMPLES

4.1 Impulse response of an RC circuit

For testing purposes the investigation of an RC circuit (fig. 9) is of principal interest. The RC circuit under consideration has the transfer function $H(p) = Y(p)/X_1(p)$ where

$$H(p) = \frac{10^3 s^{-1}}{p + 10^3 s^{-1}} \tag{44}$$

and its impulse response $h_0(t)$ is

$$h_0(t) = \delta_{-1}(t) 10^3 s^{-1} e^{-t/(10^{-3} s)}. \tag{45}$$

The frequency response $|H(j2\pi f)|$ as well as the impulse $h_0(t)$ have no sharp limitation. Therefore it is demanded arbitrarily

$$H(j2\pi f_H) = \frac{1}{40} \tag{46}$$

and

$$h_0(T_h) = \frac{1}{40} h_0(+0). \tag{47}$$

From this we get

$$T_b = 5.3\text{ms} \qquad f_H = 6.36\text{kHz}. \tag{48}$$

For Γ we again use arbitrarily $\Gamma = 10^3$ (extension of the impulse response from 6.36ms to 6.36sec).

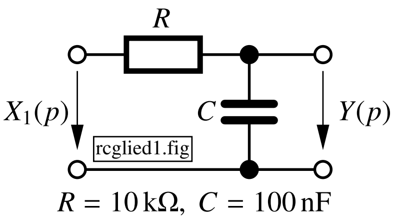

Figure 9: RC circuit, whose impulse response is to be determined by the proposed method.

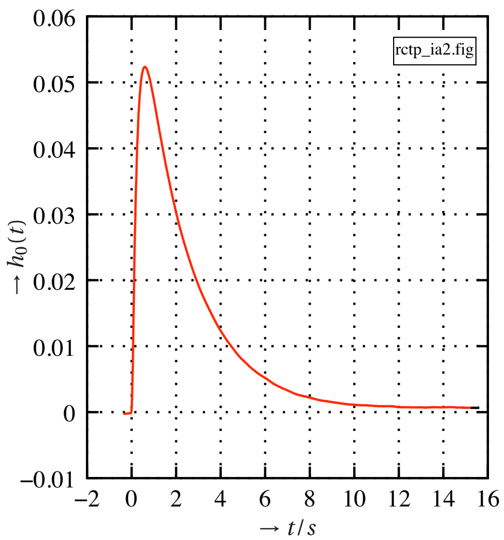

Figure 10: Measured impulse response of the RC circuit.

Further it will be requested that the function $\sin(\pi\lambda/L)/(\pi\lambda/L)$ equ. (35) should have the value 0.9, and from this

$$l_0 = \frac{L}{\lambda} = 4 \tag{49}$$

Now applies

$$f_C = 25{,}44\,\text{kHz}. \tag{50}$$

For this case it follows

$$N = \lceil \text{ld}(135{,}83) \rceil = 8 \tag{51}$$

as well as

$$L = 255. \tag{52}$$

and for λ:

$$\lambda = \left\lfloor \frac{L}{l_0} \right\rfloor = \left\lfloor \frac{255}{4} \right\rfloor = 63. \tag{53}$$

If $f = 0 \ldots f_{gp}$ is the passband and $f = f_{gs} \ldots \infty$ the stop band of the averaging low pass, we get

$$f_{gp} = 6{,}28\,\mathrm{Hz} \quad \text{and} \quad f_{gs} = 93{,}48\,\mathrm{Hz}. \tag{54}$$

In this case we can use a very simple filter.

In fig. 10 the measured impulse response of the RC circuit is drawn. It is in good agreement with the expected response. Deviations are observed at time $t = 0$, where there should occur a step. But generally speaking a step can not occur because of the low pass characteristic of the averaging filter. A low pass has a rise time greater than zero and thus no step is possible.

4.2 Impulse response of a Cauer (elliptic) filter

The next device under test was a Cauer low pass filter (also known as an elliptic low pass filter) with pass band frequency $f_D = 4.5$kHz, where the pass band attenuation is $a_D = 0.8$dB. The stop band starts at $f_S = 5$kHz, the stop band attenuation is $a_S = 55$dB.

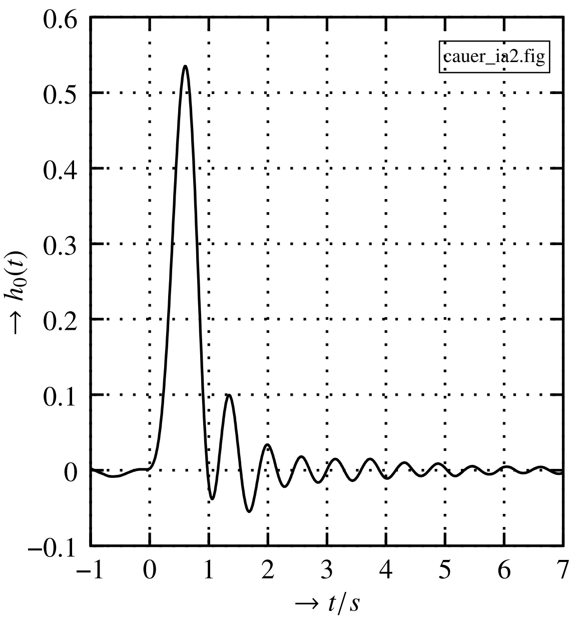

Figure 11: Measured impulse response of an elliptic filter.

The measured impulse response for this type of filter is depicted in fig. 11. It can be seen that the period duration of 8 s was chosen a bit too short, because left of the time $t = 0$ the transients of the preceding period is still present.

5 CHARACTERISTICS OF THE PROPOSED MEASURING METHOD

5.1 Preliminary remark

How exactly can the measurement circuit determine the impulse response? To answer the question it is assumed that the device under test shows a noise like interfering signal $n(t)$, whose spectral density $N_0(f)$ will be

$$N_0(f) = \begin{cases} D & \text{for } |f| \le f_H \\ 0 & \text{else.} \end{cases} \quad (55)$$

Fig. 12 shows the model of the DUT used below.

Now the signal $y'_0(t)$ is fed to the multiplier (switch). Caused by the interfering noise being now present at the low pass output appears

$$w'(t) = w(t) + n'(t), \quad (56)$$

where $w(t)$ means the periodized impulse response which results in the absence of the noise signal. $n'(t)$,denotes the interference generated by the low pass filter when excited by $n(t)$.

5.2 Definition of a figure of merit

The quality of the measurement is to be described by a figure of merit Q defined by

$$Q = \frac{\int_0^{\Gamma T_h} w^2(t)dt}{\Gamma T_h \mathrm{E}\{(n'(t))^2\}}. \quad (57)$$

Q means the quotient of useful energy and noise like energy of the output signal during time ΓT_h of the spread impulse response.

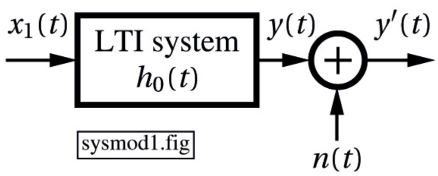

Figure 12: System model with noisy output signal.

5.3 The case $a_2 = 0$ and $b_2 = 1$

The numerator of Q equals

$$\int_0^{\Gamma T_h} w^2(t)dt = \Gamma \frac{L+1}{2Lf_C} \int_0^{T_h} h_0^2(t)dt. \tag{58}$$

For the expectation value in the denominator of equ. (57) we get[3]

$$E\{(n'(t))^2\} = D\Big[|c_0^{(2)}|^2 + |c_\lambda^{(2)}|^2 + + 2\sum_{\mu=1}^{\lambda-1} |c_\mu^{(2)}|^2\Big] \frac{2f_H}{\Gamma} \tag{59}$$

and with equ. (21)

$$\approx D\frac{L+1}{L}\left[\frac{1}{2} + \frac{\lambda}{L}\right]\frac{f_H}{\Gamma}. \tag{60}$$

Now Q is fixed:

$$Q \approx \frac{\frac{\Gamma}{f_C}\int_0^{T_h} h_0^2(t)dt}{2D\left[\frac{1}{2} + \frac{\lambda}{L}\right] f_H T_h}. \tag{61}$$

The gain of the procedure can be judged if Q is compared with the quality figure Q_r of the impulse method explained in 2.1. It is

$$Q_r = \frac{\frac{1}{f_C}\int_0^{T_h} h_0^2(t)dt}{2Df_H T_h}. \tag{62}$$

For the gain factor $q = Q/Q_r$ we have

$$q = \Gamma \frac{2}{1+2\frac{\lambda}{L}}, \quad \frac{\lambda}{L} = \frac{f_H}{f_C}. \tag{63}$$

Summarizing result: The gain factor q is proportional to the stretching factor Γ. It depends on the quotient of the cut off frequency f_H and the clock frequency f_C of the m-sequence generator.

[3] In the interest of a simple and clear final result in the following the $\sin(x)/x$ dependency of the coefficients $c_\mu^{(2)}$ will be ignored.

5.4 The case $a_2 = -1$ and $b_2 = 1$

For this case the numerator of the figure of merit is in conformity with the numerator of eqn. (61). The denominator follows from eqn. (59) together with eqn. (23). We get

$$Q \approx \frac{\frac{\Gamma}{f_C}\int_0^{T_h} h_0^2(t)\,dt}{4D\frac{\lambda}{L} f_H T_h}. \tag{64}$$

If Q normalized as above by Q_r eqn. (62), we get for q

$$q = \Gamma \frac{L}{2\lambda}.$$

For such values λ/L actually used in measuring tasks (e. g. $\lambda/L = 1/4$), q approximately equals the figure of merit of (Glg. 63).

In principle using the proposed method the figure of merit can be increased arbitrarily, but only on the expense of a growing measuring time.

6 SUMMARY

A correlation based method for measuring the impulse response of a linear and time invariant analog system is described.

It is characterized by a low technical expense: The circuitry consists only of two m-sequence generators, a clock oscillator, an electronic switch (possibly with a inverting amplifier) and a simple RC circuit (low pass).

The measurement results in a stretched and periodized form of the actual impulse response of the DUT. The acquisition and further processing of this signal is simple and easy.

It was shown that the figure of merit is governed by the stretching factor. It can be made arbitrarily large, but this is true at the expense of measuring time.

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